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UH-8047

B. E. - II (Sem. - III) (ALL) Examination

May/June – 2012

Engineering Mathematics - III

(New Course)

Time : **3** Hours] [Total Marks : **100**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **B. E. - II (Sem. - III) (ALL)**

Name of the Subject :

☛ **Engineering Mathematics - III (New)**

☛ Subject Code No. : **8 0 4 7** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.
(3) Figures on the right indicate marks.
(4) Draw the figure whenever it is necessary.

1 (a) Attempt the following : **10**

(i) State Rodrigue's formula & obtain $P_2(x)$ from it.

(ii) Define Clairaut's differential equation and obtain general solution of $\sin(y-px)=p$.

(iii) Evaluate $\Gamma\left(\frac{5}{2}\right)$.

(iv) Define Wronskian & hence evaluate Wronskian of 1,x.

(v) Define the linear differential equation of first order first degree & give its general solution.

(b) Attempt any **two** of the following : **10**

(i) Find the power series solution of the equation $y''-xy=0$, about an ordinary point x=0.

(ii) Obtain a Frobenius series solution of

$$x^2y''+xy'+\left(x^2-n^2\right)y=0$$

(iii) Find a Frobenius series solution of $9x(1-x)y''-12y'+4y=0$.

2 (a) Attempt any **two** of the following : 6

(i) $\left(x^2+1\right)\frac{dy}{dx}+\left(y^2+1\right)=0$

(ii) $\left(5x^4+3x^2y^2-2xy^3\right)dx+\left(2x^3y-3x^2y^2-5y^4\right)dy=0$

(iii) $3\frac{dy}{dx}+xy=xy^{-2}$.

2 (b) Attempt any **three** of the following : 9

(i) Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=e^x\cos 2x+\cos 3x$.

(ii) Solve $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x+In x$.

(iii) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=e^{-x}\ln x$ by method of Variation of Parameters.

(iv) Use method of reduction to find the second linearly independent solution of $y''-4xy'+4\left(x^2-2\right)y=0$; $y_1(x)=e^{x^2}$.

3 (a) Attempt any **three** of the following : 9

(i) Show that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$.

(ii) Evaluate $\int_0^1 x^4\left(1-\sqrt{x}\right)^5 dx$.

(iii) Solve $\left(\frac{dy}{dx}-2\right)^3=17e^{2x}$ by Undetermined coefficients method.

(iv) Express the polynomial $f(x)=4x^3-2x^2-3x+8$ in terms of Legendre polynomials.

(b) Attempt any **two** of the following : 6

(i) Find Fourier cosine integral of

$$f(x)=\begin{cases}\sin x, 0\le x\le\pi\\ 0,\ x>\pi\end{cases}.$$

(ii) Find Fourier sine trans of $f(x)=e^{-ax}$; $x \geq 0$ & $a>0$.

(iii) Find the Fourier integral representation of the function $f(x)=\begin{cases} x \, if \, |x|<1 \\ 0 \, if \, |x|>1 \end{cases}$.

4 (a) Attempt the following : **10**

(i) Using Linearity property of Laplace transforms find $L(\cos at)$ & $L(\sin at)$.

(ii) State second shifting theorem & using it evaluate, $L[4\sin(t-3)u(t-3)]$.

(iii) Define Periodic function; also prove that constant function is periodic function.

(iv) Define Fourier sine and cosine integral.

(v) Define one and two dimensional heat equation and hence state Laplace equation.

(b) Attempt by **two** of the following : **10**

(i) Obtain Fourier Series expansion for $f(x)=\frac{1}{2}(\pi - x)$ for $0<x<2$.

(ii) Obtain Fourier series expansion for $f(x)=\begin{cases} 0 & -\pi \leq x \leq 0 \\ x^2 & 0 \leq x \leq \pi \end{cases}$.

(iii) Find the Fourier series to represent the function $f(x)=\begin{cases} x & for \, 0 \leq x \leq \pi \\ 2\pi - x & for \, \pi \leq x \leq 2\pi \end{cases}$ and deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

5 (a) Solve any **two** following using Laplace transform technique. **10**

(i) $y'' - 3y' + 2y = 4t + e^{3t}$, $y(0)=1$, $y'(0)=-1$.

(ii) $y'' + 4y' + 3y = e^{-t}$, $y(0)=y'(0)=1$.

(iii) $y''' + 2y'' - y' - 2y = 0$, $y(0)=1$, $y'(0)=2=y''(0)$.

(b) Attempt any **two** of the following : **6**

(i) Find $L\left(\cos\sqrt{t}\right)$.

(ii) Find Laplace transform of $(t+1)^2 e^t$.

(ii) Prove that $\int_0^\infty te^{-3t}\sin t\,dt = \frac{3}{50}$.

6 (a) Solve the differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial u}{\partial x}$ for the conduction of heat along a rod without radiation, subject to the following conditions : **7**

$\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = 1$

$u(x,0) = lx - x^2$.

(ii) Attempt any one of the following : **7**

(i) A tightly stretched string of length L with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $u_0 \sin^3\left(\frac{\pi x}{L}\right)$. Find the displacement $u(x,t)$.

(ii) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

$u(0,t) = u(L,t) = 0$ for $0 < x < L$

$u(x,0) = f(x)$.